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ANALYSIS AND COMPENSATION OF STIFFNESS IN CNC MACHINE TOOL FEED SYSTEM

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Based on the elastic mechanics theory, the mathematical models of axial stiffness and torsion stiffness are constructed in accordance with single end thrust and two ends thrust. The effects of stiffness on dead band error are analyzed. With the analysis of displacement deviation induced by axial stiffness and angular displacement deflection caused by torsion stiffness, a formula to calculate the dead band error is presented. A model for Computer Numerical Control (CNC) machine tool feed system with stiffness is established. By applying computer simulation, dynamic performances, static performances and steady-state error of the system are analyzed. To reduce the effect of stiffness on the system, the feedforward control method is used to compensate stiffness. The simulation analysis shows the result that dynamic and static performances are improved, as well as steady-state error of the system is reduced by more than 58% with this approach.

Keywords: CNC machine tool; feed system; stiffness; dead band error; stiffness compensation.

1. Introduction

High-accuracy CNC milling machines are required in many manufactures as the demand for precision components and consistency of quality are growing.[1] Machining inaccuracy is one of the major limitations of the product quality in manufacturing. As an important influencing factor, stiffness can enlarge dead band error and cause actuators motion delay in CNC machine tool.[2] Much attention and effort has already been spent to analyze effects of stiffness on dynamic and static performance of system that results in significant increase of cost, calculate stiffness theoretically, and propose improvement measures.[3–5] This research focuses mainly on the stiffness by taking into account the whole system. This paper aims to construct stiffness models and analyze the stiffness effects on system performance via simulation. Finally, in terms of controlling, the compensation method for stiffness will be presented and verified.

2. Stiffness Model for Feed System

Figure 1 shows a schematic diagram of the feed system. The motor and ball screw are connected through shaft coupling. Ball screw realizes transformation of the rotary motion produced by motor into axial movement. In the feeding process, displacement deflection is inevitable because cutting force exists during machining and the drive mechanism elements are flexible. Additionally, the axial pressure or tension and load torque causes the ball screw to twist. Thus, output always lags behind input, and stability of the closed loop control system is affected greatly.

2.1. *Model of axial stiffness*

The axial stiffness reflects ability of the feed system to resist axial deformation. It refers to comprehensive pull-pressing stiffness of the drive system that includes nuts connecting table, screw ball and bearings. According to the installation method of ball screw, the feed system can be simplified to two types of model. One type features thrust at one end and simply supported at the other end, while the second type applies thrust at both ends.

Figure 2 shows the spring-mass mechanical model for the system which applies thrust at one end. Where, K_L is axial stiffness of the feed system, K_Z is axial stiffness of the bearing, K_{ZR} is axial stiffness of the bearing chock, K_S is axial stiffness of the ball screw, K_N is axial stiffness of the nut components, and K_{NR} is axial stiffness of the nut seat. In the model, springs are connected in series. Therefore, axial stiffness of the feed system K_L can be calculated as the following.

$$\frac{1}{K_L} = \sum \frac{1}{K_i} = \frac{1}{K_Z} + \frac{1}{K_{ZR}} + \frac{1}{K_S} + \frac{1}{K_N} + \frac{1}{K_{NR}} \tag{2.1}$$

The spring-mass mechanical model for the system with thrust at both ends is shown in Fig. 3(a). The symbols in Fig. 3 have the same meaning as those in Fig. 2. Normally bearings and bearing chocks at both ends are installed differently as their stiffness is not same. Strictly speaking, K_Z and K_{ZR} at different ends should be denoted as K_{Z1} and K_{ZR1}, K_{Z2} and K_{ZR2}. To carry out the analysis

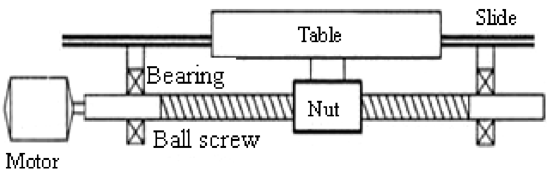
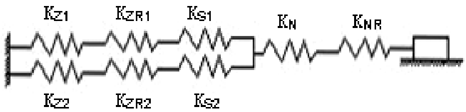
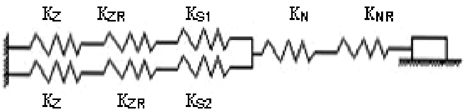


Fig. 1. Schematic diagram of the feed system.

Fig. 2. Thrust at one end.



(a) Model considering difference



(b) Model without considering difference

Fig. 3. Thrust at both ends.

better, the stiffness of bearings and bearing chocks at both ends are expressed as K_Z and K_{ZR}, while the difference between them is omitted. Therefore, spring-mass mechanical model is simplified as shown in Fig. 3(b). The model is of series–parallel hybrid.

Based on the model and elastic theory, the stiffness of the system K_L is

$$\frac{1}{K_L} = \sum \frac{1}{K_i} = \frac{1}{K_{SZ}} + \frac{1}{K_N} + \frac{1}{K_{NR}} \tag{2.2}$$

$$K_{SZ} = \frac{1}{1/K_Z + 1/K_{ZR} + 1/K_{S1}} + \frac{1}{1/K_Z + 1/K_{ZR} + 1/K_{S2}} \tag{2.3}$$

where, K_{SZ} is axial stiffness of the parallel mechanism. K_{S1} and K_{S2} are axial the stiffness of ball screw on different sides of the nut.

2.2. *Model of torsional stiffness*

Torsion stiffness is another important indicator to measure the system stiffness. It reflects the ability to resist torsion deformation, and mainly refers to the stiffness of ball screw. Thus, the torsion stiffness of the system K_θ nearly approaches the torsion stiffness of ball screw $K_{\theta S}$.

3. Deflection Due to Stiffness

Cutting force, drive torque and frictional torque causes the feed system to produce axial deflection and torsion deflection during machining. Output will be delayed and dead band error will be generated. Then, machining inaccuracy will be reduced. Cutting force and frictional torque are changed along with material performance of work piece and cutting tool, processing speed, cutting depth and other factors. Moreover, they are strongly nonlinear. So, it is complex to calculate the dead band error due to the deflection.[6] In the paper, the model with thrust at one end as shown in Fig. 2 is used to analyze the feed system. According to Eqs. (2.1) and (2.3), the deflection of the system can be expressed as shown in Fig. 4. Where K_1 is load gain which means cutting force and torque conversion coefficient. K_2 is conversion coefficient of rotation to axial displacement. θ_i is angular displacement input by drive motor. θ_o is output angular displacement. X_1 is axial displacement corresponding to the output angular. X_o is output axial displacement.

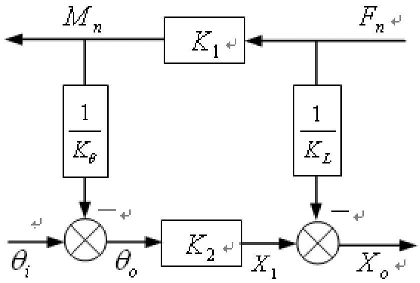


Fig. 4. Deflection block diagram.

3.1. *Displacement deflection due to axial deformation*

Displacement deflection due to axial deformation is assumed to be elastic. As seen in Fig. 4, the deflection can be calculated as follow

$$\delta_X = \frac{F_n}{K_L} \tag{3.1}$$

where, δ_X is displacement deflection. F_n is the total force along the axis. The frictional force is assumed to be made up of coulomb and viscous friction alone. Therefore,

$$F_n = F_X(s) + \text{sign}(sX_o)F_C + \mu_v sX_o + Ms^2X_o \tag{3.2}$$

where, $F_X(s)$ is cutting force. F_C is static friction, and μ_v is viscous friction coefficient. M is total mass including work piece, table, clamp and nuts.

3.2. *Angular deflection due to torsion deformation*

With the action of torque, ball screw will be twisted at a certain angle. Consequently, the angular movement at the ball screw nut θ_o is less than θ_i at the motor because of the angular twist in the ball screw. According to Fig. 4, from torsion theory, angular deflection is proportional to the load torque M_n and inversely proportional to the K_θ. So, the angular deflection can be written as

$$\delta_\theta = \frac{M_n}{K_\theta} = \frac{F_n P_h L}{2\pi\eta G I_p} \tag{3.3}$$

where, δ_θ is angular deflection of the feed system. P_h is the lead of the ball screw. η is the mechanical efficiency of the system. G is shear modulus of the ball screw. I_p is the polar moment of inertia and L is the length of the ball screw.

3.3. *Dead band error due to stiffness*

Because displacement deflection and angular deflection are inevitable, the actual motion position is different from the nominal position. Generally, the difference is called dead band error due to stiffness, and the error E_X can be given by

$$E_X = K_2\theta_i - X_o = \delta_\theta K_2 + \delta_X = F_n\left(\frac{K_1K_2}{K_\theta} + \frac{1}{K_L}\right) \tag{3.4}$$

where, E_X is the dead band error due to stiffness. K_1, K_2, K_θ and K_L are relevant to the actual motion position. So, they are dynamic. But, it is easy to calculate according to material elastic constants and position. It is important but difficult to measure. During machining, the cutting force and friction are varying with time. So, it is essential to measure them online. Traditionally, the force is monitored directly with the resistance dynamometer or force transducer. The reliability of results measured by the method is low, and the installation of force sensor is inconvenient. According to differential equations of permanent magnet DC motor drive

$$M_T = M_n + Js^2\theta_o + \mathrm{sign}(s\theta_o)M_C + \mu_\omega s\theta_o = K_T i(t) \tag{3.5}$$

where, M_T is output torque of the motor. M_C is static friction torque. μ_ω denotes viscous friction coefficient. J is total inertia of the rotating parts. K_T is torque constant of the motor.$i(t)$ is armature circuit current.

4. Simulation and Results

To analyze the stiffness effect on the system performance, position servo system is constructed ignoring the effect of friction and gap according to Fig. 4, Eq. (3.2) and Eq. (3.3). Figure 5 shows the structure of the system. x_i is input displacement and x_o is output displacement. K_l and T_l are magnification factors and time constant of phase filter, respectively, and their value are 1.11 and 0.0132. K_j and T_j are magnification factors and time constant of thyristor, respectively, and their values are 33.2 and 0.0005. K_s and T_s are magnification factors and time constant of motor armature, respectively. Their values are 0.5 and 0.0035. K_m and T_m are the gear parameters, and their values are 15.04 and 0.115, respectively. K_e, K_i, K_t and K_f are potential coefficient, current feedback coefficient, velocity

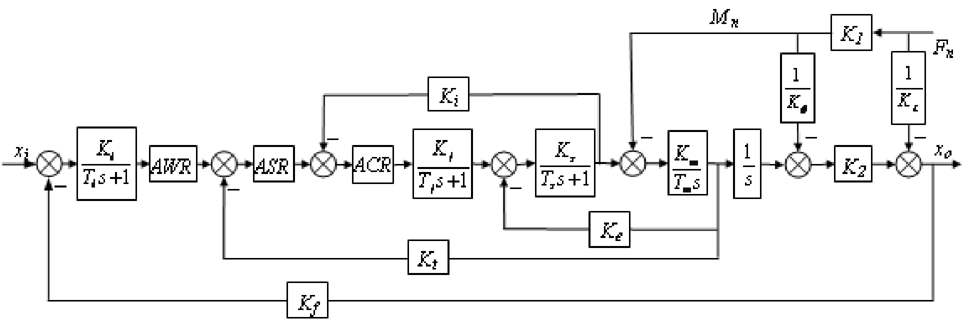


Fig. 5. Feed system block diagram.

feedback coefficient and position feedback coefficient, and their values are 0.132, 0.26, 0.01 and 2.5, respectively. The values of K_1, K_2, K_θ and K_L are 0.001, 0.07962, 3.985 and 4.24, respectively. AWR, ASR and ACR are position regulator, speed regulator and current regulator. During simulation, the parameters are adjusted repeatedly, and regulators are defined as (4.73s + 116)/(s + 50), (s + 29)/0.0048s and (s + 5)/s.

The system is simulated with Matlab/Simulink. The results are shown in Figs. 6 and 7.

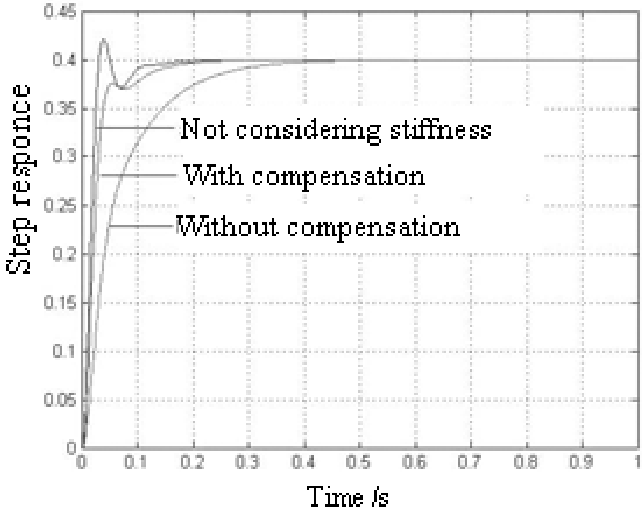


Fig. 6. Step response of the feed system.

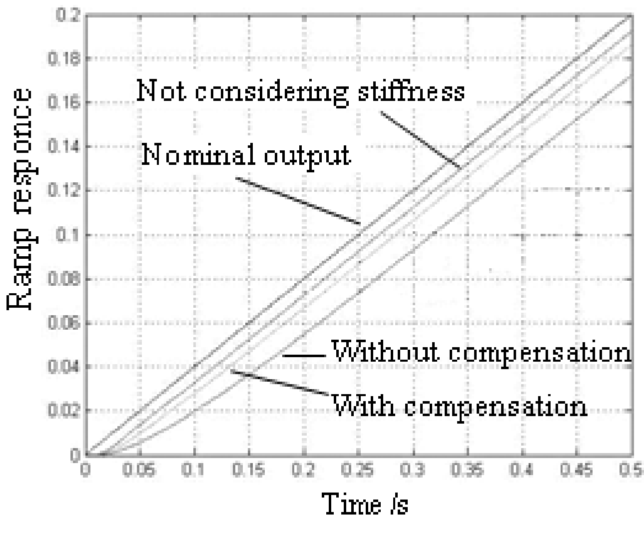


Fig. 7. Ramp response of the feed system.

Step response of the system is shown in Fig. 6. Without considering the case of stiffness and load, PID controller is designed and adjusted to ensure the rapid and stable response. When stiffness is considered, the system damping ratio ξ is increased. Then, overshoot disappears, and stability of the system is enhanced. But, adjusting time increases, and response rapidity decrease obviously. Ramp response of the system is shown in Fig. 7. Stiffness not only increases the adjusting time and the dead band error, but also makes steady state error increase by 266.7%.

5. Stiffness Compensation

The stiffness can be increased with structural improvement of the feed system through design. The costs using the method to improve stiffness are also considerably large. Stiffness compensation can reduce the effect of stiffness through improving control strategy without changing the original structure, and is increasingly coming fore as an important and effective method. Commonly, the methods to improve the stiffness include feedforward control compensation, observer compensation and so on.[7] In this paper, feedforward control method is adopted to compensate the stiffness. The element is shown in Ref. 7. The output displacement can be deduced as follow

$$y(s) = \frac{G_K(s)G_n(s)}{1 + K_m G_K(s)G_n(s)} r(s) + \frac{G_n(s)[G_K(s)G_f(s) - 1]}{1 + K_m G_K(s)G_n(s)} M(s) \qquad (5.1)$$

where, $r(s)$ and $y(s)$ are input and output respectively. $G(s)$ is transfer function of controller. $G_f(s)$ is feedforward compensation controller. $G_n(s)$ is transfer function of mechanical parts and motor. $M(s)$ is disturbance and K_J is feedback coefficient.

For the Eq. (5.1), disturbance has no effect on the system if $G_K(s)G_f(s) = 1$. The simulation results are shown in Figs. 6 and 7. The results show that the stiffness compensation improves the dynamic performance of the system, shortens adjustment time and reduces the steady state error by 58.2%. Therefore, the compensation is effective to reduce the error due to stiffness and improve the tracking performance.

6. Conclusion

As described above in detail, first, two stiffness models have been established according to the installation method of ball screw. Then analysis has been carried out on the effects of stiffness on feed system via combining the feed system model and control algorithm, on the basis of which the compensation method for stiffness is proposed and verified by simulation. Although the stiffness model is simplified, plus effects of gap and friction to the system are neglected, the results are not affected. With a view to serve further, this research predicts the cutting force, compensates the error and improves the machining accuracy significantly.

Acknowledgments

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